

On the Thermomagnetic Gas Torque for Collinear Static and Alternating Magnetic Fields*

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Results obtained within the framework of kinetic theory are presented for the part of the thermomagnetic gas torque which stems from the thermal pressure and is associated with the influence of the magnetic fields on the viscosity (Senftleben-Beenakker effect). Good qualitative agreement and a quantitative agreement within a factor of roughly 1.5 are found between the theory and the experimental data of Smith and Scott [1969].

1. Introduction

Recently the thermomagnetic torque effect¹ (SCOTT effect) for rarefied polyatomic gases has been investigated experimentally by SMITH and SCOTT^{2,3} for the case where an oscillating magnetic field collinear to the usual static magnetic field has been applied. It is the purpose of this letter to present some results obtained within the framework of kinetic theory by a simple approximation which gives a good qualitative understanding of the observed phenomena and leads to a reasonable quantitative agreement between theory and experiment.

Similar to the SENFTLEBEN-BEENAKKER effect^{4,5} (SBE) the thermomagnetic gas torque is associated with the fact that transport processes in polyatomic gases give rise to an alignment of the rotational angular momenta of the molecules. Since the alignment, in turn, influences the transport processes these are affected by an applied magnetic field due to the precessional motion of the rotational angular momenta. It seems that for a quantitative understanding of the thermomagnetic gas torque both the mechanisms for the influence of the alignment on the transport processes which have been proposed by WALDMANN⁶ and by LEVI and BEENAKKER^{7,8} have to be considered. In the first case it is assumed that due to a “surface coupling” the alignment causes a tangential flow in the presence of a magnetic

field. In the second case the effect of the alignment on the thermal pressure is considered and it has been shown that in this case the torque is determined by the SBE on heat conductivity and viscosity and by a kind of an interference effect⁸. As a simple approximation only the part of the thermomagnetic torque which stems from the thermal pressure and is associated with the SBE on viscosity is taken into account for the following treatment of the “dynamic” Scott effect. In the presence of an alternating magnetic field the torque oscillates at the same rapid frequency as the applied field. But due to the inertia of the torsion pendulum the quantity measured in the dynamic Scott effect is the time-average of the underlying oscillating torque. Accordingly, the relevant time-averaged viscosity has to be considered.

2. Viscosity in the Presence of Static and Alternating Magnetic Fields

In the viscosity problem the dominating alignment is that one which is characterized by the second rank tensor polarization of the rotational angular momenta (this is true for all gases measured so far⁵ except NH₃). Hence, the treatment of the viscosity problem can be based on two “transport-relaxation equations” which come from the “kinetic

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² G. W. SMITH and G. G. SCOTT, Phys. Rev. Letters 20, 1469 [1968].

³ G. W. SMITH and G. G. SCOTT, Phys. Rev. 188, 433 [1969].

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⁵ J. J. M. BEENAKKER and F. R. MCCOURT, Ann. Rev. Phys. Chem. 21 [1970], in print.

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⁷ A. C. LEVI and J. J. M. BEENAKKER, Phys. Letters 25 A, 350 [1967].

⁸ A. C. LEVI, F. R. MCCOURT, and J. J. M. BEENAKKER, Physica 42, 363 [1969].



equation''¹⁰⁻¹² and which couple – via collisions – the friction pressure tensor and the tensor polarization. The magnetic field occurs in the precession term of the tensor polarization. The solution of these two equations in the case of two collinear fields⁹, a static and an alternating one, is not more difficult than in the case of a static field alone (the ordinary Senftleben-Beenakker effect).

Let us denote the magnitude of the magnetic field by

$$H(1 + \beta \sin \omega t). \quad (1)$$

$$\begin{aligned} G_m = (1 + i m \omega_H / \omega_T)^{-1} I_0^2(m \beta \omega_H / \omega) \\ + 2 \sum_{l=1}^{\infty} (1 + i m \omega_H / \omega_T) \cdot [(1 + i m \omega_H / \omega_T)^2 + l^2 \omega^2 / \omega_T^2]^{-1} I_l^2(m \beta \omega_H / \omega). \end{aligned} \quad (3)$$

In Eq. (3), $\omega_H = \gamma H$ is the precession frequency of a molecule with a gyromagnetic ratio γ in the presence of a constant magnetic field with magnitude H , ω_T is the relaxation frequency of the tensor polarization which can be expressed in terms of a collision integral^{9,12,13}, and $I_l(\dots)$ is the Bessel function of the 1st kind. The quantity A_η occurring in (2) which determines the saturation value of the Senftleben-Beenakker effect can also be expressed in terms of collision integrals¹³. In the static case ($\beta = 0$) the complex viscosity coefficients $\eta^{(m)}$ are related to the coefficients η_1, \dots, η_5 of DE GROOT and MAZUR¹⁴ by

$$\eta^{(0)} = \eta_1; \quad \eta^{(1)} = \eta_3 + i \eta_5; \quad \eta^{(2)} = 2 \eta_2 - \eta_1 - i \eta_4. \quad (4)$$

The possibility to measure the magnetic-field-induced change of all 5 viscosity coefficients has recently been demonstrated by HULSMAN and BURGMANS¹⁵. For $\beta = 0$ (no alternating field) Eqs. (2), (3) yield the results obtained by MCCOURT and SNIDER¹⁶ for the SBE on viscosity.

3. Thermomagnetic Gas Torque

Due to the geometry of the experimental set-up the viscosity coefficient relevant for the Scott effect is $\eta_4 = -\text{Im } \eta^{(2)}$. Notice that this coefficient con-

Here H is the magnitude of the static field, β is the ratio between the magnitudes of the alternating and the static fields and ω is the angular frequency of the alternating field. Then the time-averaged (complex) viscosity coefficients $\bar{\eta}^{(m)}$ ($m=0, \pm 1, \pm 2$) obtained from the transport-relaxation equations are given by (η is the field-free viscosity)

$$\bar{\eta}^{(m)} / \eta - 1 = A_\eta (\bar{G}_m - 1), \quad (2)$$

with

tains the "double frequency" $2 \omega_H$. The torque in the presence of an applied field with the magnitude given by (1) is denoted by $N(H, \beta)$ and one has, within the approximation discussed above,

$$N(H, \beta) \propto \text{Im } \bar{\eta}^{(2)}(H, \beta). \quad (5)$$

With the abbreviations

$$\alpha = 2 \omega_H / \omega_T = H / H_{\max}, \quad (6)$$

where H_{\max} is the magnetic field for which the torque reaches its maximum value (in the absence of the alternating field) and

$$x = \omega / 2 \omega_H, \quad (7)$$

the relative difference between the torque with and without an alternating field is given by

$$\begin{aligned} \Delta N / N = [N(H, \beta) - N(H, 0)] / N(H, 0) \\ = I_0^2(\beta/x) - 1 + \sum_{l=1}^{\infty} g_l(x, \alpha) I_l^2(\beta/x), \end{aligned} \quad (8)$$

with

$$g_l = \frac{2(1 + \alpha^{-2})(1 + \alpha^{-2} - l^2 x^2)}{(1 - \alpha^{-2} - l^2 x^2) + 4 \alpha^{-2}}. \quad (9)$$

The case $\alpha > 1$ is of major interest since measurements of the dynamic Scott effect under this condition – as has been shown by SMITH and SCOTT³ – allow the determination of the precession frequency

⁹ S. HESS and L. WALDMANN, to be published.

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¹⁵ H. HULSMAN and A. L. J. BURGMANS, Phys. Letters **29A**, 629 [1969].

¹⁶ F. R. MCCOURT and R. F. SNIDER, J. Chem. Phys. **47**, 4117 [1967].

ω_H and thus of the gyromagnetic ratio γ . For the further discussion of Eqs. (8), (9) $\alpha \gg 1$ is assumed. Then, for $lx \approx 1$, Eq. (9) can be approximated by

$$g_l \approx \frac{1-lx}{(1-lx)^2 + \alpha^{-2}}. \quad (10)$$

Eq. (10) describes a "dispersion curve" with the "dispersion frequency" $x_l = l^{-1}$ and the "width" $(l\alpha)^{-1}$. Hence, for $\alpha \gg 1$, $\Delta N/N$ is given by a sum of dispersion curves which sit on a "background" described by $I_0^2(\beta/x) - 1$. The "height" (amplitude) between the maximum and minimum of the l -th dispersion curve is approximately given by $\alpha I_l^2(l\beta)$.

It should be stressed that the fundamental dispersion frequency is $\omega = 2\omega_H$ rather than $\omega = \omega_H$

which is actually the first subharmonic. The next subharmonics, which have not yet been resolved experimentally, are $\omega = \frac{2}{3}\omega_H$ and $\omega = \frac{1}{2}\omega_H$.

The occurrence of the double frequency $2\omega_H$ is not due to a nonlinear effect but due to the fact that a 2nd rank tensor polarization precessing about a magnetic field may already assume an equivalent position after a rotation by 180° . This explains qualitatively why no higher dispersion frequencies ($3\omega_H, 4\omega_H, \dots$) have been found experimentally [in accord with Eqs. (8), (9)].

In Fig. 1 $\Delta N/N$ has been plotted as function of x for $\alpha = 6.4$ and $\beta = 0.81, 1.07, 1.42, 1.88$. A comparison with the measured curves (for NO) presented in Fig. 11 of Ref. ³ shows a good qualitative agreement. For a quantitative comparison between theory and experiment the "amplitude" $A(2\omega_H)$ of the dispersion peaks at $\omega \approx 2\omega_H$ (cf. Fig. 1) and the ratio of the amplitudes $A(2\omega_H)/A(\omega_H)$ of the peaks at $\approx 2\omega_H$ and $\approx \omega_H$ (cf. Fig. 1) are listed in Table 1 for several values of α and β . The calculated values have been obtained according to Eq. (8) where the first 4 Bessel functions have been taken into account. In view of the approximations made and of the fact that Eqs. (8), (9) do not contain any adjustable parameter the agreement found between the calculated and the measured values can be considered to be quite satisfactory.

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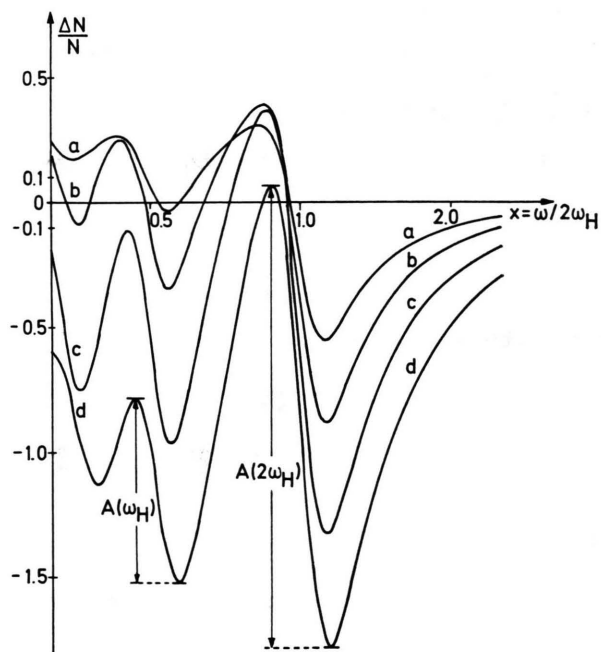


Fig. 1. The relative difference between the torques with and without an oscillating field $\Delta N/N$ as function of the reduced frequency variable $x = \omega/2\omega_H$. The parameters α and β chosen in Eqs. (8), (9) are: $\alpha = 6.4$ for all 4 curves plotted, and $\beta = 0.81, 1.07, 1.42, 1.88$ for the curves a, b, c, d, respectively. The meaning of the amplitudes $A(2\omega_H)$ and $A(\omega_H)$ is shown for curve d.

α	β	$A(2\omega_H)$		$A(2\omega_H)/A(\omega_H)$	
		calc.	exp.	calc.	exp.
6.4	0.81	0.8	0.5	3.0	—
6.4	1.07	1.3	0.8	2.1	—
6.4	1.42	1.7	1.1	2.0	2.0
6.4	1.88	1.8	1.2	2.5	2.7
7.9	1.52	2.3	1.5	2.0	1.8

Table 1. Comparison between the calculated and measured values for the "amplitudes" $A(2\omega_H)$ and the ratio $A(2\omega_H)/A(\omega_H)$. For the meaning of $A(2\omega_H)$, $A(\omega_H)$ see Fig. 1.